Fuglede's spectral set conjecture on cyclic groups
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Fuglede's conjecture (1974) states that a bounded measurable subset in $\mathbb{R}^d$ accepts an orthogonal basis of exponential functions (i.e. it is spectral) if and only if it tiles the space with a discrete set of translations. This conjecture turned out to be false by Tao's counterexample in 2003. Using Tao's ideas, counterexamples in finite Abelian groups such as $\mathbb{Z}_N^d$ can be lifted to counterexamples in $\mathbb{R}^d$, thus shifting the interest on this conjecture to this setting in recent years. This has been successful for $d>2$, but the conjecture is still open for $d=1,2$.

Some recent results in the cyclic group setting will be presented in this talk, which are connected to the work of Coven-Meyerowitz and Laba on tiling subsets of $\mathbb{Z}$, as well as the structure of vanishing sums of roots of unity. This is partially joint work with Mihalis Kolountzakis.